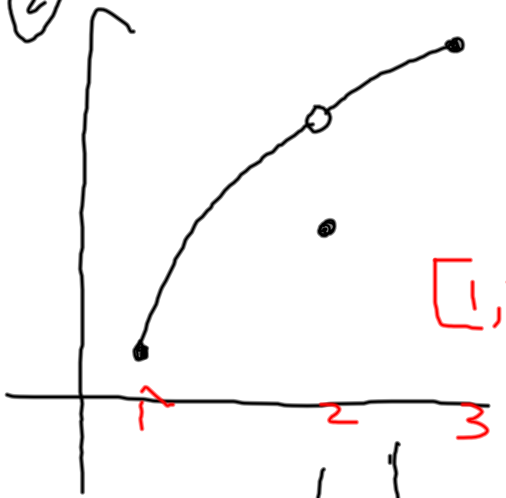
Homework 2.5

2



[1,3]

(1,3)

[1,2]

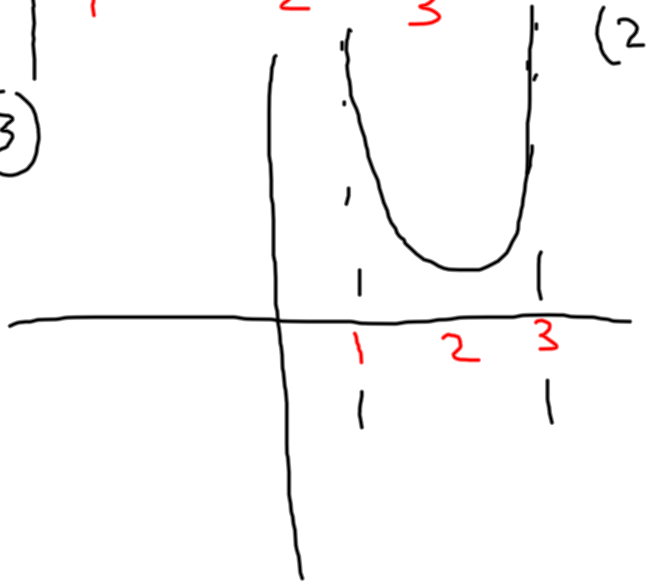
(1,2)

[1,2)

[2,3]

(2,3)

3



Pd 2: 2010-09-20

2-3, 7-9

18

20

24-25, 26,

27-28

7) f, g cont.

$f(2) = 1$

$\lim_{x \to 2} [f(x) + 4g(x)] = 13$

a) $g(2) =$

b) $\lim_{x \to 2} g(x) =$

$$\lim_{x \to 2} [f(x) + 4g(x)] = 13$$

so $\lim_{x \to 2} f(x) + 4 \lim_{x \to 2} g(x) = 13$

$$f(2) + 4 \lim_{x \to 2} g(x) = 13$$

$$1 + 4 \lim_{x \to 2} g(x) = 13$$

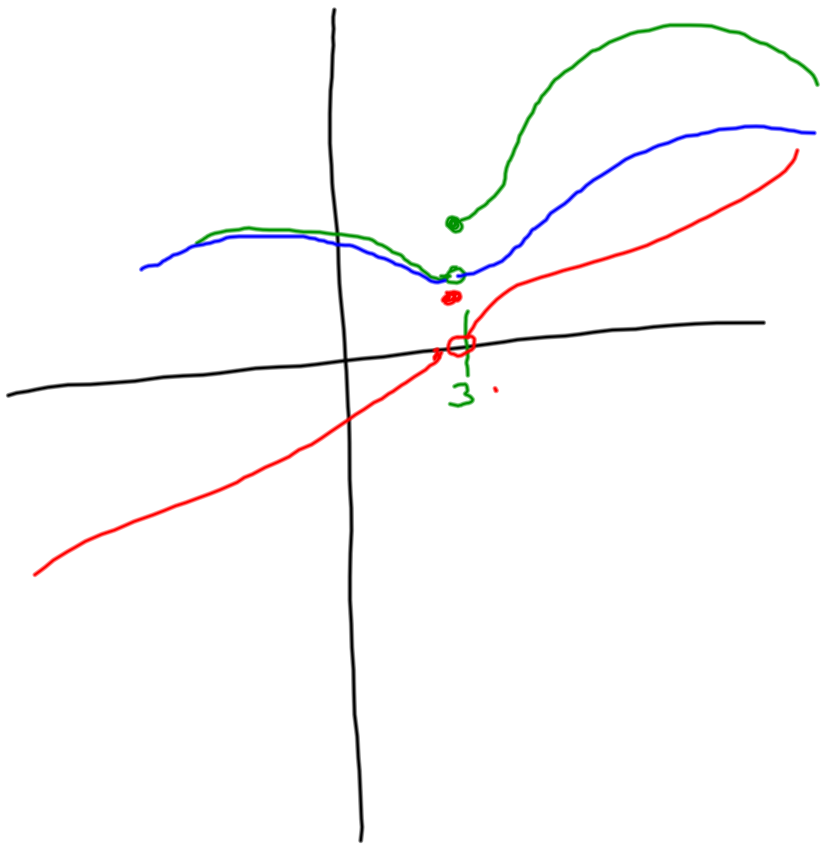
$$g(2) = \lim_{x \to 2} g(x) = 3$$

9) a) f is cont. everywhere except $x=3$ (at which pt it <u>is</u> cont fr. right)

b) f has a 2 sided limit @ $x=3$, but not cont.

c) f is NOT cont $x=3$
but if $f(3)=0$ it becomes cont.
$f(3)=1$

d) f cont $[0,3)$
def on $[0,3]$
not cont on $[0,3]$



18) $f(x) = \dfrac{3x+1}{x^2+7x-2}$

$$x^2+7x-2=0$$

$$x = \frac{-7 \pm \sqrt{49-4(1)(-2)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{57}}{2}$$

20) $f(x) = \dfrac{5}{x} + \dfrac{2x}{x+4}$

$$= \frac{\quad}{x(x+4)}$$

$$x = 0$$

$$x = -4$$

24) $f(x) = \begin{cases} \dfrac{3}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{3}{x-1} = $ DNE ($+\infty$, $-\infty$)

candidate for discont

f is not cont:

★ possibly where pieces are "glued" together

$x = 1$?

★ possibly where rational f^n "pieces" are not defined

$\dfrac{3}{x-1}$ not defined at $x=1$

but $x=1$ isn't in domain of this rule

f(x) is NOT cont @ $x=1$

25) a) $f(x)=\begin{cases} 7x-2, & x\le 1 \\ kx^2, & x>1 \end{cases}$

check f(x) cont @ x=1

a) $\lim_{x\to 1} f(x) \to \lim_{x\to 1^+} kx^2 = k(1)^2 = k$

$\lim_{x\to 1^-} f(x)$

$= \lim_{x\to 1^-} 7x-2 = 7(1)-2 = 5$

$k=5$

ⓑ $f(1) = 7-2 = 5$

b) $f(x)=\begin{cases} kx^2, & x\le 2 \\ 2x+k, & x>2 \end{cases}$

$x=2$?

a) $\lim_{x\to 2} f(x) \to \lim_{x\to 2^+} 2x+k = 2(2)+k = 4+k$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} kx^2 = k(2)^2 = 4k$

$4k = 4+k$

$3k = 4$

$k = \frac{4}{3}$ makes f(x) continuous

ⓑ f(2) exists

$f(2) = kx^2\Big|_{x=2} = \frac{4}{3}x^2\Big|_{x=2}$

$f(2) = \frac{4}{3}(2)^2 = \frac{16}{3}$

ⓒ $\lim_{x\to 2} f(x) = 4\left(\frac{4}{3}\right) = \frac{16}{3}$?

$4 + \frac{4}{3} = \frac{16}{3}$

26) $f(x) = \dfrac{1}{\sqrt{x-2}}$ domain AT MOST is $[2, \infty)$

I. fn has to be defined

$\sqrt{x-2}$ undefined?

$x-2<0$

$x<2$ undefined

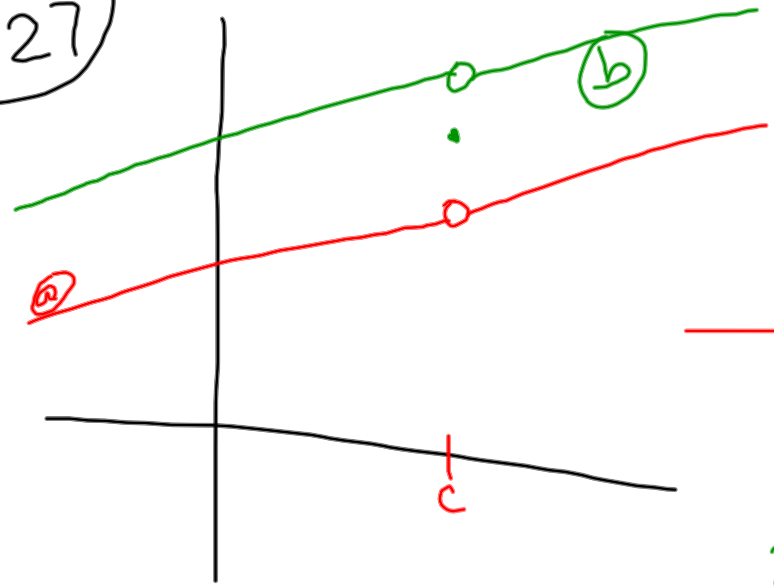
throw away $x=2$

(can't div by 0)

$f(x)$ is cont

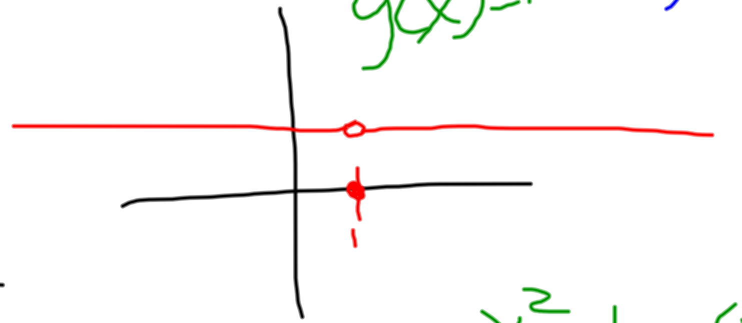
on $(2,\infty)$

27)



28b) $g(x) = \begin{cases} 1, & x > 1 \\ 0, & x = 1 \\ 1, & x < 1 \end{cases}$

$g(x) = 1$



28b $f(x) = \dfrac{x^2-1}{x-1} = \dfrac{(x-1)(x+1)}{(x-1)}$

$f(1) = \lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$= \lim\limits_{x\to 1} x+1 = 2$

Homework 2.5

2010-09-20

Pd3

18 (twice)

23-24, 25

26, 27, 28

19-22

(18) $f(x) = \dfrac{3x+1}{x^2+7x-2}$

find disc.

$$x^2+7x-2=0$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{(7)^2-4(1)(-2)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{57}}{2}$$ discontinuity

19)

$$f(x) = \frac{x}{|x|-3}$$

disc:
where denominator $= 0$

$$|x|-3 = 0$$

when $|x| = 3$

$$x = 3$$

or

$$x = -3$$

$1 + \frac{4}{x}$

$g(x) = \frac{x-1}{x-1}$

20) $f(x) = \frac{5}{x} + \frac{2x}{x+4}$

$f(x)$ disc:

$$x = 0$$

$$x = -4$$

$$f(x) = \frac{}{x(x+4)}$$

$$x = 0, -4$$

21) $|x^3-2x^2|$

disc:

$|x^3-2x^2|=0$

don't waste time

$x=0,2$

always continuous

22) $\dfrac{x+3}{|x^2+3x|}$

disc

$|x^2+3x|=0$

$x^2+3x=0$

$x=0,-3$

23) $f(x) = \begin{cases} 2x+3, & x \le 4 \\ 7 + \frac{16}{x}, & x > 4 \end{cases}$

potential discontinuities

☆ where pieces are glued

→ chk $x=4$

☆ chk within each piece

☆ chk $2x+3$

always cont

☆ $7+\frac{16}{x}$?

chk $X=0$

but only applies $x>4$

so ok here

chk $x=4$

☆ $\lim_{x\to 4} f(x)$ exist?

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} 7+\frac{16}{x}$

$= 7+\frac{16}{4} = 11$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 2x+3 = 11$

$\therefore \lim_{x\to 4} f(x) = 11$

☆ $f(4)$ exist? yes $f(4)=11$

☆ $f(4) = 11 = \lim_{x\to 4} f(x)$ $\therefore$ cont.

$\therefore f(x)$ is continuous everywhere

24) $f(x) = \begin{cases} \frac{3}{x-1} &, x \neq 1 \\ 3 &, x = 1 \end{cases}$

$$\lim_{x\to 1} f(x)$$

possible disc:

$x = 1$ (because piecewise fns glued together there)

$$= \lim_{x\to 1} \frac{3}{x-1} = DNE$$

$$\lim_{x\to 1^+} \frac{3}{x-1} = +\infty \qquad \lim_{x\to 1^-} \frac{3}{x-1} = -\infty$$

$f(x)$ disc @ $x = 1$

25) a $f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$

Find k so that f^n is continuous.

cond 1

$\lim_{x \to 1} f(x) = ?$

$\lim_{x \to 1^-} f(x) =$

$\lim_{x \to 1^-} 7x-2$

$= 7(1)-2 = 5$

$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1^+} kx^2$

$= k(1)^2 = k$

$k = 5 \Rightarrow$ 2 sided lim exists

Cond 2

$f(1) = 7(1)-2 = 5$

Cond 3

$5 = 5$

so

$f(1) = \lim_{x \to 1} f(x)$

$\therefore k = 5$ makes $f(x)$ cont everywhere

25b $f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x+k, & x > 2 \end{cases}$

Find k to make f(x) continuous

$\lim_{x\to 2} f(x) = ?$

$\lim_{x\to 2^-} f(x)$ $\qquad$ $\lim_{x\to 2^+} f(x)$

$= \lim_{x\to 2^-} kx^2$ $\qquad$ $\lim_{x\to 2^+} 2x+k$

$= 4k = 4+k$

$4k = 4+k$

$3k = 4$

$k = \frac{4}{3}$

2ND cond

$f(2) = \left(\frac{4}{3}\right)2^2$

$= \frac{16}{3}$

3rd

does $f(2) = \lim_{x\to 2} f(x)$?

$\frac{16}{3}$ $\qquad$ $\frac{16}{3}$

they are =.

$\lim_{x\to 2} f(x) = 4\left(\frac{4}{3}\right) = \frac{16}{3}$

$4 + \frac{4}{3} = \frac{16}{3}$

$\therefore k = \frac{4}{3}$ makes f(x) continuous everywhere

26) consider $\dfrac{1}{\sqrt{x-2}}$

on what intervals is this continuous?

1) what is domain?

$\Rightarrow$ can't $\sqrt{\text{neg}\#}$

$x-2 \geq 0$

$x \geq 2$

$\Rightarrow$ can't divide by zero

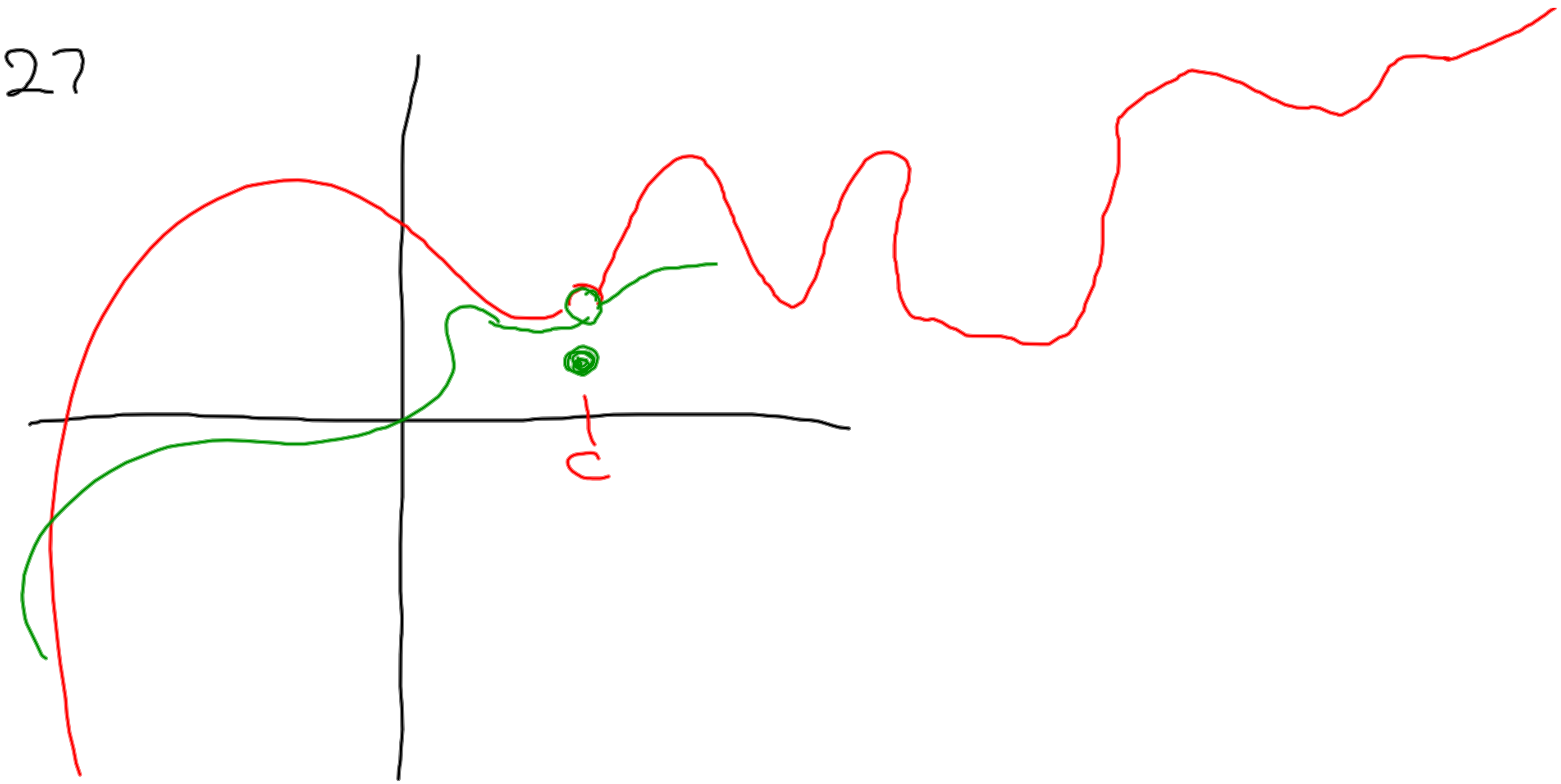
$x > 2$

2) given domain is $x > 2$

f^n is cont everywhere in domain.

$$\lim_{x \to \infty} \frac{1}{\sqrt{x-2}} = 0$$

27



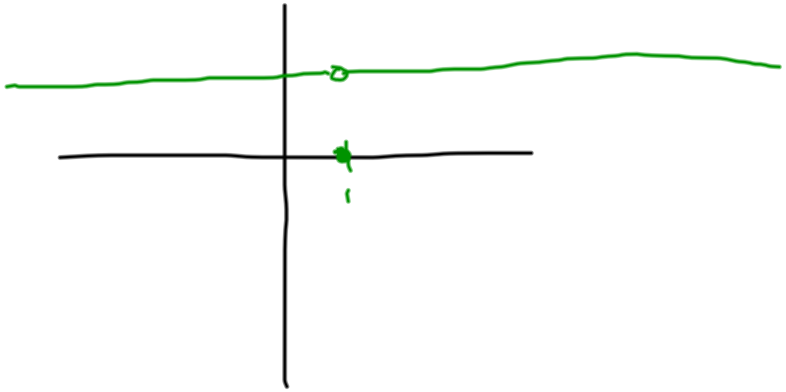
28] $f(c) = \lim_{x\to c} f(x)$

28bii

$$g(x) = \begin{cases} 1 & x<1 \\ \cancel{3}\ 1 & x=1 \\ 1 & x>1 \end{cases}$$

$g(x)=1$ on

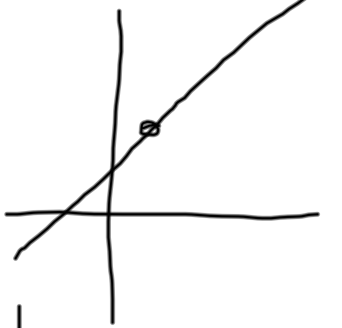
removable disc @ $x=1$



(b)

$$\frac{x^2-1}{x-1}$$

removable dis: @ $x=1$



fix it by $f(1)=\lim_{x\to 1}\frac{x^2-1}{x-1}$

$$= \lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)}$$

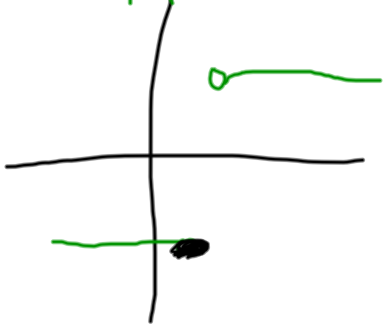
$$= \lim_{x\to 1} x+1 = 2$$

2.5/39, 40, 42

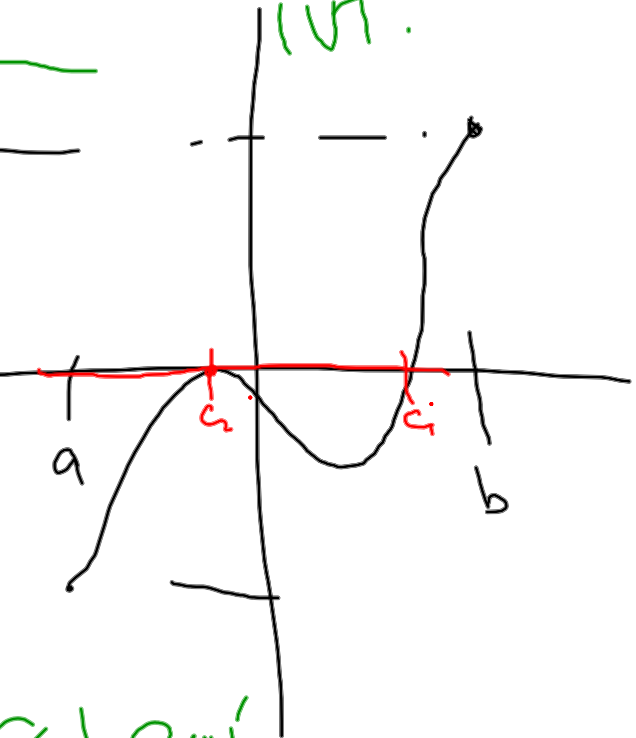
39)

* $f(x)$ def $[a, b]$
* $f(a), f(b)$ opposite signs
* $f(x) = 0$ has no solution

Ans:



IVT:

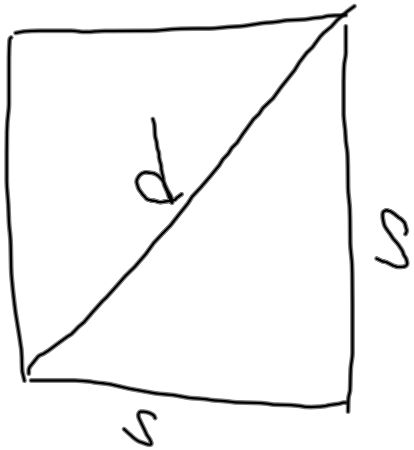


$\therefore$ $f(x)$ CAN'T be continuous

40) IVT

$\exists$ square w/ diag length betw. r & $2r$

where Area $= \frac{1}{2}\pi r^2$



$$d^2 = s^2 + s^2$$

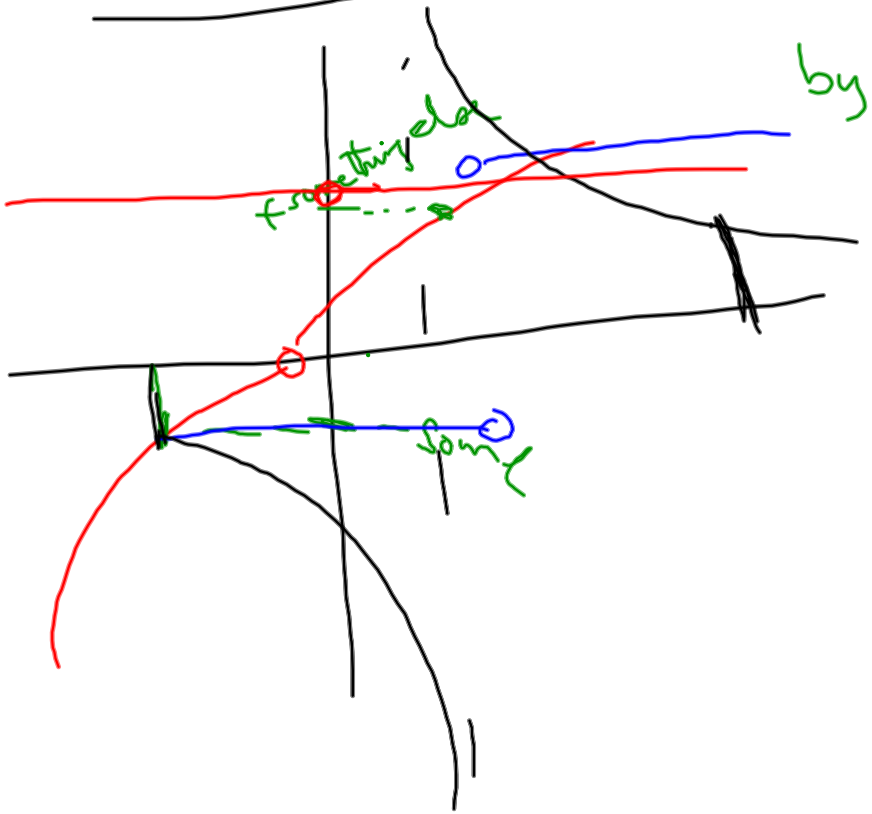
$$d^2 = 2s^2$$

$$\frac{d^2}{2} = s^2 = A$$

2.5/39 fn f that is * defined on a closed interval

* values at endpoints have opposite signs

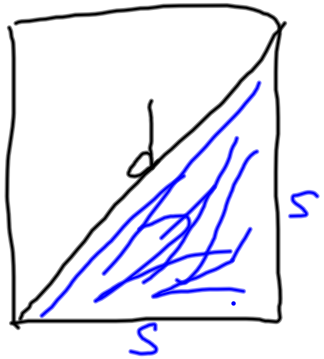
* $f(x)=0$ has No solution



by the IVT

f can't be continuous

40)

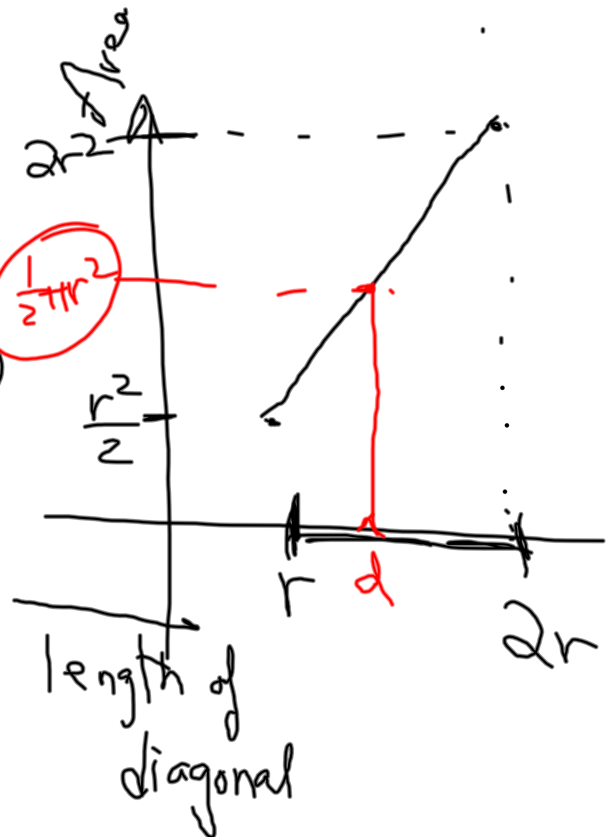


$r < d < 2r$

$$\text{Area} = s^2$$
$$s^2 + s^2 = d^2$$
$$2s^2 = d^2$$
$$s^2 = \frac{d^2}{2}$$

Area of square

$$= \frac{1}{2}\pi r^2$$



$$\text{Area}_{2r\boxtimes} = \frac{(2r)^2}{2}$$
$$= \frac{4r^2}{2} = 2r^2$$

4.1) $\exists$  $j < r$ whose volume is $=$ 

"there exists"

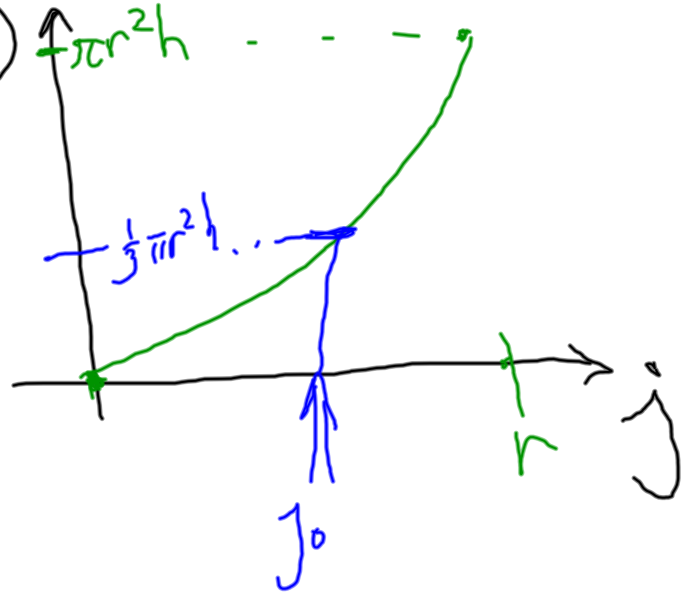
Use IVT

$V_{cyl} = \pi (\text{radius})^2 (\text{height})$

$V = \pi j^2 h$

$V_{cone} = \frac{1}{3} \pi (\text{radius})^2 (\text{height})$

$\frac{1}{3} \pi r^2 h$

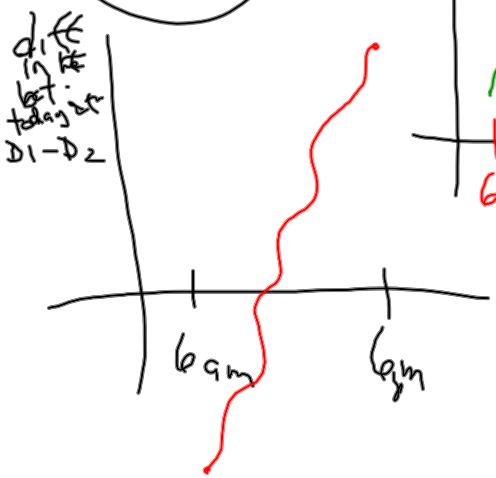


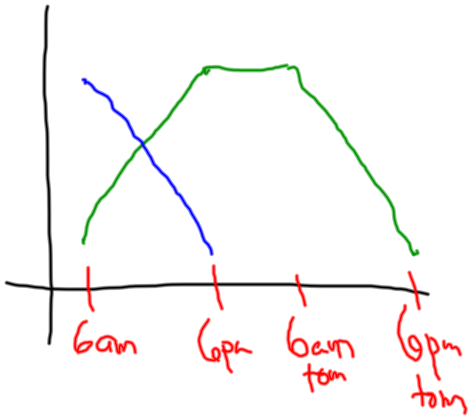
A monk leaves the bottom of the Mt @ 6:00AM
walking to the temple at the top
arriving @ 6pm.

stays over night :
leaves at 6AM arriving at the bottom @ 6pm.

Show ~~there~~ was a pt he was
at the same exact time.

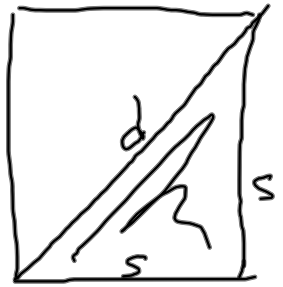
#52





40

NT:



$r < d < 2r$

Area of square

$= \frac{1}{2}\pi r^2$

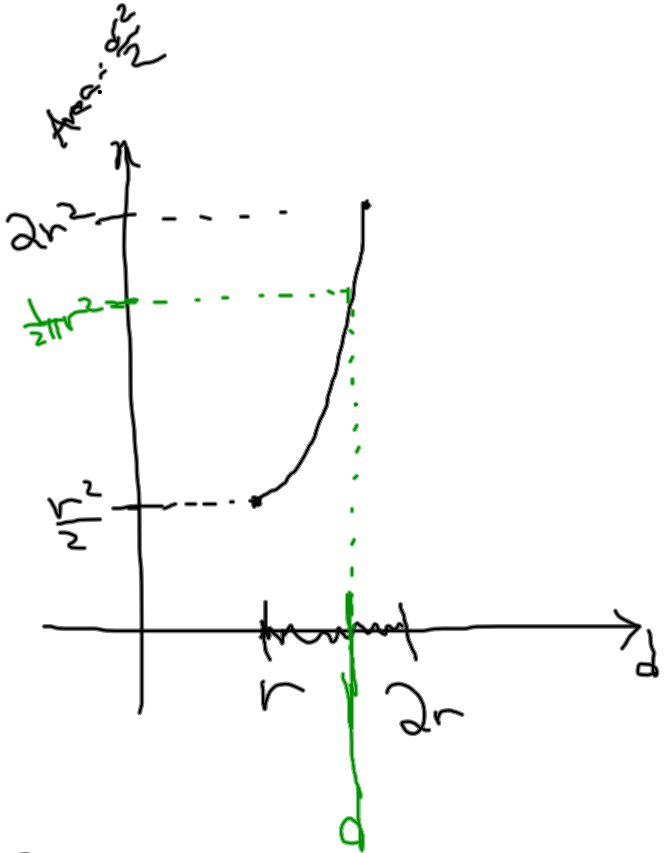
$s^2 + s^2 = d^2$

$2s^2 = d^2$

$s^2 = \frac{d^2}{2}$

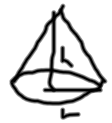
Area $= s^2$

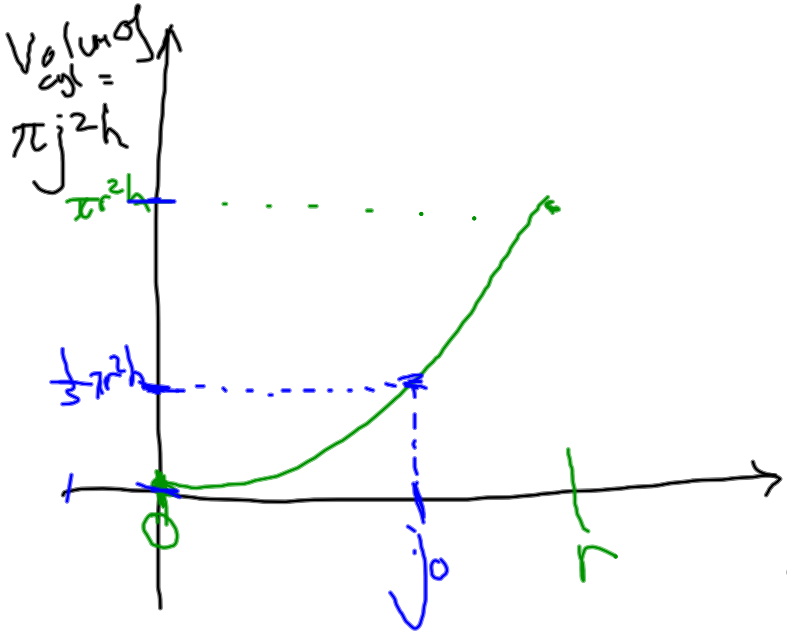
$\frac{(2r)^2}{2} = \frac{4r^2}{2} = 2r^2$



41) $\exists$ $[j < r]$ where V_{cyl} $(\pi j^2 h)$ $= V_{cone}$ $\left(\frac{1}{3}\pi r^2 h\right)$

there exists





j (radius of cylinder)

42) $x^3-4x+1=0 \quad [1,2]$

$f(x)=x^3-4x+1$

$f(1)=-2$

$f(2)=1$